ESTABLISHMENT OF A NONUNIFORM MODE OF FLUIDIZATION WITH A UNIFORM DISTRIBUTION OF THE FLOW OF THE COOLING MEDIUM

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Using the principle of the mechanical stability of a system, a criterion for the disturbance of the uniformity of fluidization is obtained and the initial properties of the nonuniform fluidization state of a layer of fine particles close to a distributed grating (the porosity of the dense phase, and the concentration and dimensions of the drops which appear) are considered.

The fluidization systems used in practice are usually nonuniform and can be regarded as a combination of a uniform "dense" phase with a high particle concentration and "bubbles" of the fluidizing medium propagating in it, containing practically no particles. The problem of the characteristics of the bubbles and of the dense phase and also what part of the medium flows through the dense phase of the layer and what part in the phase of the bubbles is the essential problem which arises in the simulation and design of chemical reactors and other apparatus based on a fluidized bed (see, e.g., [1]).

An effective solution of this problem involves, firstly, surmounting the difficulties which arise when analyzing the motion, increase, and rupture of a single bubble in a uniform layer, which becomes considerably worse when taking into account the hydrodynamic interaction and coalescence of bubbles under constrained motion. Secondly, the properties of actual nonuniform layers depend to a considerable extent on the dimensions and concentration of the "initial" bubbles which occur in the lower part of the bed directly over the distributing grating. If there is a jet feed of the medium through a nozzle or an opening in the perforated grating the initial characteristics of the bubbles formed are determined mainly by the flow rate of the jet in the elementary jets and depend only slightly on the physical parameters of the granular bed [2-5]. A quite different situation occurs in the case of a uniform feed of the fluidizing medium, e.g., using a porous grating, when the fact of the appearance of bubbles and also their initial properties depend completely on the physical parameters of the medium and the particles. An analysis of this situation is given in this paper.

The occurrence of nonuniformities is usually connected with instability of the uniform fluidized state with respect to small random perturbations. However, such an instability is characteristic both for coarsely dispersed beds, fluidized by gases, which are always nonuniform, and for layers of fine particles fluidized by dropping liquids, in which the transition to a nonuniform fluidization mode is generally not observed. In addition, the rate of increase can be estimated only for very small perturbations (see, e.g., [6]). Hence, the applicability of the above considerations to the problem of establishing a nonuniform mode is problematical at the present time. Under these conditions it is natural to use certain principles to solve this problem which have a fairly general and fundamental character, e.g., the well-known extremal principles of classical mechanics and the thermodynamics of irreversible processes. If even these principles are difficult to prove in the case of the very complex system considered, they can be introduced as a postulative basis as possessing a high degree of physical reliability.

The first attempts to use this approach to investigate nonuniform fluidized systems were made in [7, 8]. The main idea of these publications is that in the presence of an excess flow rate of the fluidizing medium the granular bed expands so that its potential energy in the gravitational field is a minimum (an approach developed by Doichev [9-11]), by which a criterion was obtained of the establishment of a nonuniform fluidization mode and by which the correctness of this principle and the initial assumptions were confirmed phenomenologically by comparing its conclusions with the conclusions which follow from other criteria of the onset of the nonuniform mode [12-14], proposed in the framework of other physical models. However, in [9-11] it was assumed a

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priori that the porosity of the dense phase is identical with the porosity of the granular bed in the dense-packing state.* More general approaches based on the introduction for the fluidized system of a local potential and its use to analyze a number of ideas formulated in the nonlinear theory of thermodynamic instability [15] are also known. In this paper we use considerations similar to those put forward in [9-11].

We will represent the system as spatially uniform in the sense that the size and concentration of the bubbles are approximately the same in the volume it occupies. This means neglecting the effects of the boundaries of the layer and also all the processes by which the bubbles arise as a result of exchange between the fluidizing medium and the dense phase, coalescence and bursting. Hence we will only investigate first of all the nonuniform fluidized state that occurs. If these processes are not pronounced, as occurs, for example, for the low layers with low nonuniformity, this assumption may represent a good approximation to the description of the actual fluidized bed as a whole. Note that a similar assumption, also made in [9-11]; is widely used for simplification even when it is quite untrue (see, e.g., [1, 16, 17]).

The particles are assumed to be small so that the hydraulic resistance of the particles of the dense phase with respect to the flow of the fluidizing medium is linear with respect to the rate of filtering. Finally, for simplicity, we will assume that the volume concentration of the bubble phase φ is small compared with unity. This enables us to assume that the rate of rise of a group of bubbles differs from the rate of rise of a single bubble of the same dimensions in the unlimited dense phase by an amount proportional to φ, and also enables us to assume that the total effect of the bubbles on the flow of the fluidizing medium can be obtained by summing the effects of the individual bubbles as if they were the same. In addition, this assumption enables us to neglect the downward motion of the dense phase which neutralizes the transfer of particles upwards in the wake zones of the rising bubbles, the velocity of which is proportional to the product of φ by the small ratio of the volume of this zone to the sum of the volumes of the zone and the bubble [1].

We will characterize the state of the idealized bed considered using the diameter D of the sphere equal in volume to the bubble, and also by the values of the effective porosity ε of the bed as a whole and the porosity ε_d of its dense phase. The volume fraction of the bubble and the relative expansion of the layer are related to ε and ε_d by the equations

$$\varphi = \frac{\varepsilon - \varepsilon_d}{1 - \varepsilon_d}, \quad \frac{H}{H_*} = \frac{1 - \varepsilon_*}{1 - \varepsilon}, \tag{1}$$

which follows from the condition of conservation of the granular material in the bed.

The rate of filtering of the fluidizing medium in the dense phase (which we will assume to be static on the average) can be written in the form†

$$u_d = \frac{1}{k_1} \frac{\varepsilon_d^3}{1 - \varepsilon_d} \frac{d_1 - d_0}{\nu d_0} g (2a)^2, \tag{2}$$

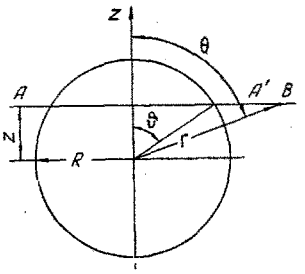

Fig. 1. Sketch to explain the derivation of the formula for the average flow of fluidizing medium.

*Obviously, the class of system of the particle–continuous-medium type, to which the principle of minimum potential energy can be applied, must of necessity be limited. In [7-11] only truly fluidized systems, in which the particles are in a suspended state, i.e., their weight is completely compensated by the hydraulic forces of interaction with the relative flow of the fluidizing medium, are taken as belonging to this class. This eliminates from consideration, for example, states in which a considerable part of the medium "breaks through" in randomly forming channels so that the granular material settles and fluidization ceases. A similar limitation is naturally imposed on the system considered in this paper.

†Strictly speaking, the relation which applies to mobile particles of the dense phase (e.g., the Richardson–Saki formula), and not to a fixed layer, the hydraulic resistance of which is considerably higher than that of a uniform fluidized bed, cannot be used. However, all the above formulas are approximate, and the use of an expression of form (2) simplifies the comparison with the data obtained by other workers and with that given in [9-11].

corresponding to the well-known Karman–Kozen ($k_1 = 180$) and Ergan ($k_1 = 150$) equations.

The rate of rise of the bubbles can be represented in the form*

$$u_b = u_{b0}(1 + \alpha\varphi), \quad u_{b0} = k_2 (gD)^{1/2} \left[\frac{(1-\varepsilon_d)(d_1 - d_0)}{(1-\varepsilon_d) d_1 + \varepsilon_d d_0} \right]^{1/2}, \tag{3}$$

where, according to Taylor's theory, $k_2 = 0.711$. Obviously, Eq. (3) only makes sense if the bubble in fact represents a "macroscopic" form, i.e., $D/2a \gg 1$.

We will consider the effect which an individual bubble has on the flow rate of the liquefying medium through a horizontal plane AA', which intersects the bubble as shown in Fig. 1. In the system of coordinates connected with the bubble, for the vertical component v_z of the velocity of the medium at the point B inside the dense phase we have [4]

$$v_z = v_r \cos\theta - v_\theta \sin\theta = \left(\frac{R}{r}\right)^3 (u_b + 2v) \left(\cos^2\theta - \frac{1}{2}\sin^2\theta\right) - (u_b - v), \quad v = u_d/\varepsilon_d, \quad R = D/2.$$

The plane AA' intersects the bubble at different levels z with equal probability; using the fact that $\cos^2\theta = z^2/r^2$, and averaging over all possible z, it is easy to obtain the expression

$$\langle \cos^2\theta \rangle = \frac{1}{R}\int_0^R \frac{z^2}{r^2}\, dz = \frac{1}{3}\left(\frac{R}{r}\right)^2, \quad \langle \sin^2\theta \rangle = \frac{2}{3}\left(\frac{R}{r}\right)^2.$$

Hence, the first term on the right side of the equation for v_z generally makes no contribution to the average vertical component of the velocity, which is equal to $-u_b + v$. In the laboratory system of coordinates the average velocity and flow of the medium in the dense phase equal v and u_d, respectively, i.e., are the same as their values when there are no bubbles. The average "deficit" of the flow through the dense phase due to the fact that part of the plane AA' passes inside the bubble is obviously equal to $\pi R^2 \langle \sin^2\vartheta \rangle u_d n$, where n is the numerical concentration of intersected bubbles in the plane AA'. The average flow of fluidizing medium through the area lying inside the bubble, on the basis of the results obtained in [4], can be written as $\pi R^2 \cdot \langle \sin^2\vartheta \rangle (u_b + 3u_d)n$.. Taking into account the fact that by definition $\pi R^2 \langle \sin^2\vartheta \rangle n = \varphi$, we obtain for the average flow in the layer

$$u = (1-\varphi) u_d + \varphi (u_b + 3u_d) = (1 + 2\varphi) u_d + \varphi u_b. \tag{4}$$

A relation of this type was already assumed in [19], and then used by many authors for a modification of the two-phase theory of fluidization. However, for large bubbles surrounded by clouds of closed circulation, it has been subjected to criticism, e.g., in [1], where instead of $u_b + 3u_d$ in (4) u_b was used (this inaccuracy is also characteristic for [9-11]). As follows from the analysis given above, there is no basis for this criticism.

No theory of the constrained motion of coarse bubbles exists at the present time. Hence, to estimate the coefficient α in (3), which from general physical considerations should not depend either on D or on the properties of the dense phase, we will use experimental data, which, according to [4, 18], are satisfactorily correlated by the equation

$$u_b = u - u_d + u_{b0}$$

*Considerations given in [4], according to which the velocity of the bubble in an actual system with continuous supply of the fluidizing medium exceeds u_b from (3) by an amount $u - u_d$, are erroneous. The conclusions in [4] are based on a comparative analysis of two systems of similar gas bubbles of the same volume concentration Φ in a dropping liquid: 1) uniformly filling the whole volume of liquid, which is ensured by continuous supply of the gas downwards with an average velocity U, and 2) of the final cloud of bubbles overlapping the whole cross section of the column. It is clear that in the first case the liquid on average is static, while the velocity of the bubbles $U_1 = U_b$, $\Phi U_1 = U$. In the second case we obtain from the balance relation for conservation of volume that the liquid inside the cloud is displaced downwards with velocity $U_f = -\Phi(1-\Phi)^{-1} U_2$, where U_2 is the velocity of the bubbles relative to the laboratory system of coordinates in this case. It is obvious that $U_2 = U_b + U_f = U_1 - \Phi(1-\Phi)^{-1} U_2$, so that $U_2 = (1-\Phi) U_1$ and $U_1 - U_2 = \Phi U_1 = U$. Hence, the difference between the observed velocities of the bubbles in the systems considered will be as stated in [4] and as confirmed by the experiments of Niklin [18] quoted in [4]. But this difference is not because the velocity of the bubble exceeds the value U_b, associated with the motion in the static liquid in the first case, but on the contrary is due to a reduction in the velocity of the bubbles compared with U_b by an amount U in the second case. The inadequacy of the suggestions put forward in [4] is pointed out in a note to p. 143 in [6].

for fairly dense bubbles, when $u_{b0} \gg u_d$. Using the last inequality and relation (4) in this equation for u, we obtain, apart from second-order terms in φ, an equation for u_b, and comparing it with (3), an expression for α. In this way we have

$$u_b \approx u_{b0}(1+\varphi), \quad \alpha \approx 1. \tag{5}$$

Introducing the dimensionless parameters

$$N = \frac{u}{u_*}, \quad T = \frac{1}{k_1 k_2}\left[\mathrm{Ar}\,(\varkappa - 1)\left(\frac{2a}{D}\right)\right]^{1/2}, \quad \varkappa = \frac{d_1}{d_0}, \tag{6}$$

$$\mathrm{Ar} = g\,(\varkappa - 1)\,\frac{(2a)^3}{\nu^2}$$

and using (1)-(3) and (5), we can rewrite relation (4) in the form

$$N' = \frac{\varepsilon_*^3}{1-\varepsilon_*} N = \frac{\varepsilon_d^3(1+2\varepsilon-3\varepsilon_d)}{(1-\varepsilon_d)^2} + \frac{1}{T}\,\frac{\varepsilon-\varepsilon_d}{1-\varepsilon_d}\left(1+\frac{\varepsilon-\varepsilon_d}{1-\varepsilon_d}\right)\left[\frac{(1-\varepsilon_d)(\varkappa-1)}{(1-\varepsilon_d)\varkappa+\varepsilon_d}\right]^{1/2}. \tag{7}$$

According to the principle of maximum mechanical stability of the expanded fluidized bed used in [9-11], the initial nonuniform state, if it in general arises, is such that the value of the flow from (4) or of the fluidization number from (7) is accurately equal to the similar value for a uniform layer of the same particles, fluidized by the same medium having the same porosity ε. Assuming that an expression of the form (2) also holds for the rate of filtering of the medium in a uniform layer we obtain

$$T = \frac{\varepsilon-\varepsilon_d}{1-\varepsilon_d}\left(1+\frac{\varepsilon-\varepsilon_d}{1-\varepsilon_d}\right)\left[\frac{(1-\varepsilon_d)(\varkappa-1)}{(1-\varepsilon_d)\varkappa+\varepsilon_d}\right]^{1/2}\left[\frac{\varepsilon^3}{1-\varepsilon} - \frac{\varepsilon_d^3(1+2\varepsilon-3\varepsilon_d)}{(1-\varepsilon_d)^2}\right]^{-1}. \tag{8}$$

Equations (7) and (8) can be regarded as a system of two equations in three unknown quantities: T, ε, and ε_d. This system is incomplete and hence we cannot determine these quantities in the form of unique functions of the physical parameters (occurring in the expressions for Ar and $\varkappa$ in (6)) and the system parameter – the fluidization number N. In [9-11] this difficulty is circumvented using the two-phase theory of fluidization by making the arbitrary assumption $\varepsilon_d = \varepsilon_*$. However, there is considerable evidence (see, in particular, [20-23]) that the true distribution of the flow of the liquefying medium between the bubbles and the dense phase in a number of cases deviates considerably from the requirements of the two-phase theory. When analyzing mass- and heat-exchange processes between a medium and particles and when designing catalytic reactors with a fluidized bed and certain other apparatus this difference can sometimes be neglected. Nevertheless, its role is extremely large when estimating different transfer coefficients in the dense phase of a layer connected with pulsating (pseudoturbulent) motion of the particles, since the intensity of the latter depends very much on ε_d in the region of ε_*, and when $\varepsilon_d \to \varepsilon_*$ this motion is generally degenerate [24].

We will here assume as the main hypothesis that in a system with a given fluidization number, containing bubbles of a given size, the flow of fluidizing medium is distributed between the bubble and dense phase so that the porosity ε is a minimum, i.e., the center of gravity of the expanded layer occupies the lowest possible position. Differentiating (7) with respect to ε_d with the conditions N' = const and T = const and assuming $\partial\varepsilon/\partial\varepsilon_d = 0$, we obtain an equation for determining the function $\varepsilon_d = \varepsilon_d(\varepsilon)$ which has the following formal form:

$$\partial N'/\partial\varepsilon_d = 0, \quad T = \mathrm{const}. \tag{9}$$

It is easy to see that this equation is equivalent in accuracy to the equation which would be obtained using a quite different hypothesis, viz., that for specified T and ε the flow of fluidizing medium is distributed between the phases in such a way that the dissipated energy of the flow is a minimum. In fact, the power dissipated per unit area of transverse cross section of the layer is equal to $u\Delta p$, and the pressure drop Δp is independent of the distribution of the flow of the medium. Hence, the condition that the energy dissipation should be a minimum is equivalent to the condition that the flow u from (4) for the parameter N' from (7), considered as a function of ε_d, should be a minimum, which again leads to equation (9). Calculating the derivative in (9) using (7) and substituting the relation for T, following from (8), into the expression obtained, we obtain

$$\begin{aligned}(\varepsilon-\varepsilon_d)\,\varepsilon_d^2[(3-\varepsilon_d)(1+2\varepsilon-3\varepsilon_d) - 3\varepsilon_d(1-\varepsilon_d)] = [\varepsilon^3(1-\varepsilon_d)^2 - \\ -\varepsilon_d^3(1-\varepsilon)(1+2\varepsilon-3\varepsilon_d)]\left\{\frac{1+2\varepsilon-3\varepsilon_d}{1+\varepsilon-2\varepsilon_d} + \frac{\varepsilon-\varepsilon_d}{2(1-\varepsilon)[\varkappa(1-\varepsilon_d)+\varepsilon_d]}\right\},\end{aligned} \tag{10}$$

defining ε_d in the form of a function of ε and $\varkappa$. It is clear that $\varepsilon_* \leqslant \varepsilon_d \leqslant \varepsilon$; if the root of Eq. (10) lies to the left of ε_* on the numerical axis, we must take $\varepsilon_d = \varepsilon_*$, and if it lies to the right of ε then $\varepsilon_d = \varepsilon$.

Analysis shows that apart from the obvious root $\varepsilon_d = \varepsilon$, Eq. (10) has one real root in the interval $(0, \varepsilon)$,

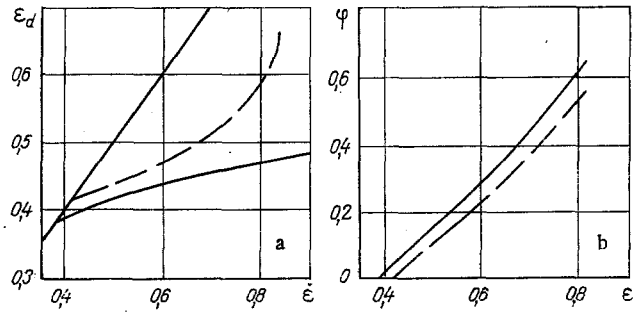

Fig. 2. Dependence of the porosity of the dense phase ε_d (a) and the volume concentration of the phase of the bubbles φ (b) on the effective porosity of the layer as $\varkappa \to \infty$ (the continuous curves), and $\varkappa = 2$ (the dashed curves).

if ε exceeds a certain critical value depending on $\varkappa$, which can be obtained by expanding (10) in series in terms of the small parameter $\delta = \varepsilon - \varepsilon_d$ in the region of the point $\varepsilon_d = \varepsilon$, and requiring that as δ increases the left side of (10) should increase more rapidly than the right. As $\varkappa \to \infty$ this critical value is approximately equal to 0.343; it increases somewhat as $\varkappa$ decreases.

The results of a numerical solution of Eq. (10) for ε_d are illustrated in Fig. 2; it can easily be shown by a direct check that this root (10) in fact corresponds to the lowest position of the center of gravity of the layer or to minimum energy dissipation in it. Figure 2 also shows the dependence on ε of the volume concentration φ of the bubble phase, following from (1). The dependence of ε_d and φ on $\varkappa$ ceases to be important when $\varkappa \gg 1$. At the beginning of fluidization, i.e., for values of ε which differ only slightly from ε_*, the root of Eq. (10) is less than ε_*, so that in accordance with the main postulate of the two-phase theory of fluidization we have $\varepsilon_d = \varepsilon_*$. However, for large ε the value ε_d may differ considerably from ε_* and from ε. If the value of ε_* for a specified granular bed is greater than a certain critical value, which depends on $\varkappa$, the layer becomes nonuniform immediately on passing through the value $N = 1$ – the fluidization number; otherwise the appearance of nonuniformities persists until N or ε reach certain values, somewhat exceeding unity or ε_*. This effect is particularly pronounced for beds of solid particles in dropping liquids. On the whole, the point of view expressed in [25] is confirmed regarding the absence of a basic difference between beds fluidized by gases and dropping liquids.

Using the porosity ε_d obtained above, we find T from (8) in the form of a function of ε and $\varkappa$, which enables the relative dimensions of the initially appearing bubbles to be determined from (6):

$$\frac{D}{2a} = \frac{(\varkappa - 1)\,\mathrm{Ar}}{(k_1 k_2 T)^2} = \gamma\,(\varkappa - 1)\,\mathrm{Ar}. \tag{11}$$

The dependence of T and γ on ε is shown in Fig. 3 (in the estimates we took $k_1 = 150$ and $k_2 = 0.711$).

In different modifications of the two-phase theory of fluidization it is usually assumed that [26]

$$u = Ku_* + \varphi u_b, \tag{12}$$

where K is a certain unknown coefficient. Comparing (12) with (4) and using (1) and (2), we obtain

$$K = \frac{1 + 2\varepsilon - 3\varepsilon_d}{1 - \varepsilon_d} \cdot \frac{1 - \varepsilon_*}{1 - \varepsilon_d} \left(\frac{\varepsilon_d}{\varepsilon_*}\right)^3. \tag{13}$$

The dependence of K on ε is illustrated in Fig. 4, whence it can be seen that K may considerably exceed unity, in qualitative agreement with experimental facts, including those given in [26]. We emphasize that (13) only relates to the initial nonuniform state considered and, consequently, has limited applicability to actual systems in which the relation between the flows of fluidizing medium in the dense and bubble phases changes as a result of exchange between the bubbles and the dense phase, coalescence, and breakdown of the bubbles.

Substituting (8) into (7), we obtain

$$N' = \frac{\varepsilon^3}{1 - \varepsilon}, \tag{14}$$

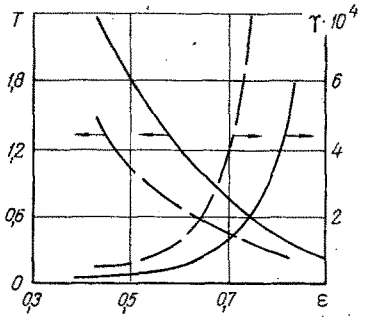

Fig. 3

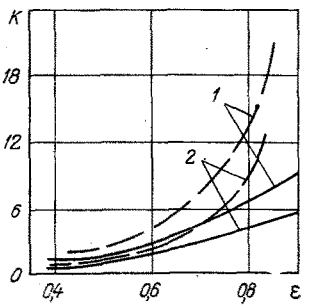

Fig. 4

Fig. 3. Dependence of the parameters T and γ on ε as $\varkappa \to \infty$ and $\varkappa = 2$ (the continuous curves and the dashed curves, respectively).

Fig. 4. Dependence of the parameter K on ε as $\varkappa \to \infty$ and $\varkappa = 2$ (the continuous curves and the dashed curves, respectively) for $\varepsilon_* = 0.35$ (1) and 0.40 (2).

so that a trivial recalculation enables us to obtain the dependence of the quantities shown in Figs. 2-4 in the form of functions of N' or of N and ε_* for different $\varkappa$. Note that the effect of the parameter $\varkappa$ in the region $\varkappa > 2$-5 on the shape of these curves is not very great, so that for $\varkappa \gg 1$ they can be considered as universal.

If u does not exceed u_* by very much, we have $\varepsilon = \varepsilon_d + x$, $x \ll \varepsilon_d$ and $\varepsilon_d \approx \varepsilon_*$. Using this in (8), we obtain the critical value for the parameter T at the instant when fluidization begins:

$$T_* = \frac{1-\varepsilon_*}{\varepsilon_*^2(3-4\varepsilon_*)}\left[\frac{1-\varepsilon_*)(\varkappa-1)}{(1-\varepsilon_*)\varkappa+\varepsilon_*}\right]^{1/2}. \tag{15}$$

The granular bed becomes "macrononuniform" for $u = u_* + 0$ if the quantity $D/2a$ is fairly large. Using values of the numerical coefficients $k_1 = 150$ and $k_2 = 0.711$ when determining T in (6), and taking, as in [9-11], as the characteristic "boundary" value $D/2a = 25$, we obtain from (15) a criterion for the establishment of the macrononuniform mode immediately when the layer changes into the fluidized state

$$N_f > (N_f)_{cr}'' = 533\,\frac{1-\varepsilon_*}{\varepsilon_*^2(3-4\varepsilon_*)}, \quad N_f = \left[\mathrm{Ar}\left(\varkappa + \frac{\varepsilon_*}{1-\varepsilon_*}\right)\right]^{1/2}. \tag{16}$$

(Here we have specially used the notation employed in [9-11].) The right side in (16) slightly differs from that in [9-11], which is due to the use there of the incorrect relation for u instead of (4) and the value $k_1 = 180$ instead of $k_1 = 150$ (when using the first value the numerical coefficient in (16) should be 640, as in [9-11]). Qualitatively, however, all the conclusions drawn in [9-11], are confirmed on the whole, and the criterion (16) corresponds to experiment and to other previously proposed criteria to the same extent as the criterion in [9, 11]. Satisfactory results are also obtained by comparing (16) with the criteria proposed later in [27, 28].

In conclusion, we will discuss some limitations of the above theory. First of all, the results obtained for the structure of the initial state only make sense if fairly large bubbles are formed, to which the above equations can be applied. For this it is necessary for, say, condition (16) to be satisfied. Further, for large ε_d, when the dense phase is considerably rarefied, it is generally not clear whether bubbles of the type usually considered exist. Hence, the model may also not describe the disappearance of the nonuniform state when there is a transition from the fludized bed to a system with diluted phase. Nevertheless, for beds fluidized by a dropping liquid ($\varkappa < 10$), for $\varepsilon > 0.8$ the value of ε_d rapidly begins to approximate to ε, which also causes a corresponding reduction in φ (see Fig. 2). Finally, we made certain simplifying assumptions, of which the most important was the assumption of the linearity of the hydraulic resistance of the particles of the dense phase with respect to the velocity of filtering of the fluidizing medium and the smallness of the volume concentration of the bubble phase. It is easy to see that the breakdown of these assumptions does not introduce new difficulties of a fundamental character, apart from the problem of determining the dependence of u_b on φ.

NOTATION

Ar, Archimedes criterion introduced in (6); a, radius of the particles; D, equivalent bubble diameter; d_0 and d_1, densities of the fluidizing medium and the material of the particles; g, acceleration due to gravity; H, height of the layer; K, coefficient in Eq. (12) for the two-phase theory of fluidization; k_1 and k_2, numerical coefficients in (2) and (3); N and N', usual and modified fluidization numbers; n, numerical concentration of the bubbles; R, bubble radius; r, radial coordinate in Fig. 1; T, a parameter introduced in Eq. (6); u, rate of filtering of the fluidizing medium; u_b and u_{b0}, bubble velocity under conditions of restricted flow and the velocity of a single bubble; v, velocity of the medium in the spaces between particles; z, level of intersection of the bubble by the horizontal plane in Fig. 1; α, a coefficient which describes the effect of restriction of the flow on the bubble velocity; γ, a coefficient in Eq. (11); $\delta = \varepsilon - \varepsilon_d$; ε, porosity; $\varkappa$, ratio of the densities in Eq. (6); ν, kinematic viscosity of the medium; θ and ϑ, polar angles in Fig. 1; φ, volume concentration of the bubble phase. Indices: d and the asterisks, dense phase and state of dense packing (initial fluidization) respectively; and angular brackets, averaging over z.

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